

## NEW ASPECTS OF NUCLEAR LEVEL SPACING DISTRIBUTIONS

Gert H. Rohr

Commission of the European Communities, Joint Research Centre - Geel-Establishment,  
Central Bureau for Nuclear Measurements, 2440 Geel, Belgium.

**Abstract:** Neutron resonances of the  $4n$  target nuclei  $^{28}\text{Si}$ ,  $^{32}\text{S}$ ,  $^{40}\text{Ca}$ ,  $^{52}\text{Cr}$  and  $^{96}\text{Zr}$  are studied. According to the independent particle model the excited states of these nuclei are created in one or maximum two collisions and are the best candidates for studying the properties of nucleon-nucleon interaction in nuclei. It is shown that the doorway resonances and resonances originating from the same doorway state result in a Gaussian-like nearest level spacing distribution. The strong energy correlation between resonances is assigned to the nucleon-nucleon interaction and contradicts the independent particle picture. The almost equally spaced doorway states are important, since they fulfil in place of equidistant single particle states the rather simple assumption of the Bethe level density formula. Furthermore, they explain the steps in the level density systematics, which is a pre-assumption for the prediction of level spacings for nuclei which are not measurable.

(Level spacing distribution, nucleon-nucleon interaction in nuclei, doorway states, nuclear models)

Introduction

According to present understanding the spacings of neutron resonances are distributed like a Wigner function in agreement with Bohr's compound nucleus model, indicating a chaotic motion of the nucleons inside the nucleus<sup>1</sup>. In the independent particle model it is assumed that the interaction of the incoming neutron with the nucleons of a target nucleus proceeds stepwise via two-body collisions<sup>2</sup>. For this regular motion of nucleons a Poisson-like distribution is expected.

It may be asked whether the effect of the two-body forces acting in these collisions shows up in the level spacing distributions. Such an effect should be predominantly observable for resonances which are created in one collision only. The best candidates for the detection of such resonances are medium-light nuclei with  $A < 38$  and closed shell nuclei<sup>3</sup>. However this study could be extended to two-step collisions for nuclei where the doorway states have a small spreading width and a small fragmentation into  $3p-2h$  states.

Nature seems to offer such examples for target nuclei with  $4n$  nucleons. The small spreading width of the doorway state causes discontinuities in the level spacing at threshold energies where the  $3p-2h$  states become energetically possible. For neutron energies near this threshold the fragmentation of the doorway state into  $3p-2h$  states is small (~ 15%) so that it becomes possible to separate doorway and  $3p-2h$  resonances<sup>4</sup>. In total nine doorway resonances of the target nuclei  $^{28}\text{Si}$  and  $^{32}\text{S}$  have been assigned. More doorway resonances for nuclei below the  $3p-2h$  threshold at  $A=38$  cannot be allocated since there are less than four  $s$ -wave resonances available in the literature<sup>5</sup>. But  $3p-2h$  resonances created in two collisions can also be included in the study of two-body interaction if the spreading width is smaller than the level spacing of the doorway states, preventing a mixing of the  $3p-2h$  energy levels from different doorway states.

The goal of this contribution is to study the nearest level spacing distribution of simple neutron excited states in  $4n$  target nuclei.

The first part is concerned with the selection of doorway resonances in  $^{28}\text{Si}+n$  and  $^{32}\text{S}+n$ . In the second part an example for selected  $3p-2h$  resonances in  $^{52}\text{Cr}+n$  is given. In the third part  $s$ -wave resonances from  $^{40}\text{Ca}+n$  and  $^{96}\text{Zr}+n$  are included for the nearest level spacing distribution which are not allocated in respect of the number of collisions.

The nearest level distribution for these selected simple excited resonances deviates from a Wigner as well as from a Poisson distribution and indicates a much stronger energy correlation between resonances, resulting in an almost equidistant spacing. The consequences of this unique property of particle excited states for nuclear physics will be discussed in the conclusions.

Selection of doorway resonances

In order to interpret the structure in the level density systematics the level density of doorway states has been calculated and compared with the observed level density of compound resonances<sup>3</sup>. There is an agreement between both sorts of data for nuclei with  $A < 38$  and closed shell nuclei; therefore the resonances of the nuclei at neutron separation energy are doorway resonances created by a collision of the incoming neutron with one of the nucleons of the nucleus resulting in a  $2p-1h$  state. The increase in the level density beyond  $A=38$  indicates the next collision where the doorway states are fragmented into  $3p-2h$  states. Additional steps in the experimental level density are observed at  $A=69$  and  $A=94$  indicating the fragmentation into  $4p-3h$  and  $5p-4h$  states respectively. The discontinuity of the level density at  $A=38$  is obtained with the increase in the single particle level density at Fermi surface energy. A similar effect should be observable for nuclei below  $A=38$  by studying the resonance structure as a function of the excitation energy.

In doing so total neutron cross section measurements performed over a large energy range with high resolution are needed and have been performed recently with the electron Linac in Geel.

The results for s-wave resonances of  $^{32}\text{S}+n$  are presented in Fig.1. The reduced neutron width is plotted as a function of the resonance energy up to 1.7 MeV<sup>4</sup>. A brief inspection of the distribution of the resonance energies points to an accumulation of resonances starting beyond 1.0 MeV, which indicates the expected branching from doorway states into 3p-2h states. The level spacing below and beyond 1 MeV changes by a factor of three from 102 keV to 315 keV respectively.

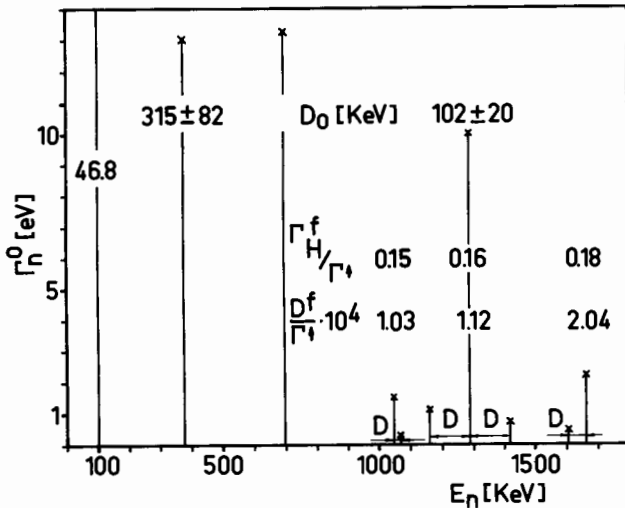


Fig. 1 s-wave resonance energies and reduced neutron widths for  $^{32}\text{S}+n$  (inserts see text).

The fragmentation of the doorway states is rather small, since if we assume that the resonances at 1048, 1290 and 1659 keV are doorway resonances and the small 3p-2h resonances close to the doorway resonance belong to the same doorway structure, the doorway parameters show a regular energy dependence. The results are given in the insert of the figure, where  $\Gamma^f$  is the sum of the reduced width for the fine structure resonances and  $\Gamma^\dagger$  is the decay width of the doorway state. The ratio  $\Gamma^f/\Gamma^\dagger$  is a measure of the fragmentation and has been calculated for each doorway structure. Its value increases monotonically with neutron energy. The same holds for the ratio  $D^f$  (average level spacing for fine structure resonances) and  $\Gamma^\dagger$ . The last ratio indicates a level repulsion effect. The splitting of the fine structure resonances is proportional to the strength of the doorway resonance and assigns the repulsion effect to the collision process. Another striking effect is the strong correlation of the energies of doorway resonances. Any selection of two nearest spacings of doorway states determines the level spacing within the error based on the calculation of all six resonances, assuming a Wigner distribution.

Fig.2 presents the s-wave resonance parameters of  $^{28}\text{Si}+n$  up to an energy of 3 MeV<sup>4</sup>. Nine resonances are observed and they are grouped in three doorway structures. Also here the fragmentation is small and increases monotonically with the neutron energy but only if we leave out the isobaric analog state<sup>6</sup>. In studying nucleon-nucleon interaction we have to exclude this resonance since it is fed mainly by electro-magnetic interaction. The repulsion effect

is also present, but a larger energy dependence of  $\Gamma^f/\Gamma^\dagger$  is observed since the doorway resonances are far above the 3p-2h threshold. Also here the equal spacing of doorway states is observed.

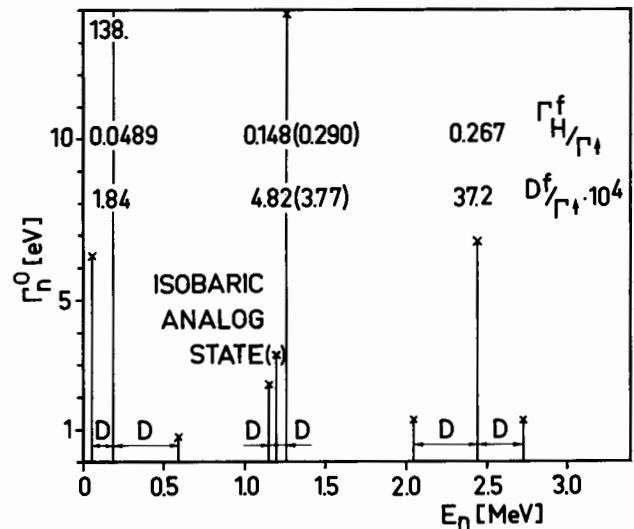


Fig. 2 s-wave resonance energies and reduced neutron widths for  $^{28}\text{Si}+n$  (inserts see text).

In Fig.3 the nearest level distribution of  $^{28}\text{Si}+n$  and  $^{32}\text{S}+n$  doorway resonances is plotted as an histogramme. For comparison the Wigner and Poisson distributions are included. The experimental spacing distribution is narrower and is represented in this figure by a Gaussian function. The limitation of spacings to the interval  $0.78 \leq D/\langle D \rangle \leq 1.2$  indicates a strong mutual repulsion effect of levels. Although the number of seven spacings is rather small, the distribution certainly deviates from the Wigner distribution since the probability of finding a spacing inside this interval, assuming a Wigner distribution, is 0.288 and consequently the probability for seven spacings is  $P_7 = (0.288)^7 = 0.016\%$ .

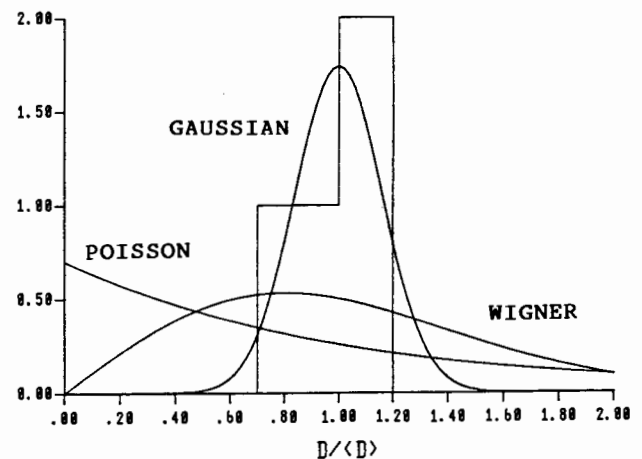


Fig. 3 Distribution of the nearest level spacings for  $^{28}\text{Si}+n$  and  $^{32}\text{S}+n$  resonances (7 spacings) compared with Poisson, Wigner and Gaussian functions.

Example for selection of 3p-2h resonances

As explained in the introduction the nearest level distribution of doorway states will favourably reflect the dynamics of nucleons in nuclei. However the more complicated states may lose this property due to the special excitation process in nucleon-nucleon interaction. The additional states created in the second collision are obtained by branching of doorway states individually into 3p-2h states up to about 8 to 10 levels as indicated schematically in Fig 4. In a similar way each 3p-2h state will branch into several 4p-3h states in the third collision etc. In order to prevent a mixing in energy of nuclear states which are originating from different doorway states, the spreading widths  $\Gamma \downarrow$  of the doorway states have to be smaller than their level spacing.

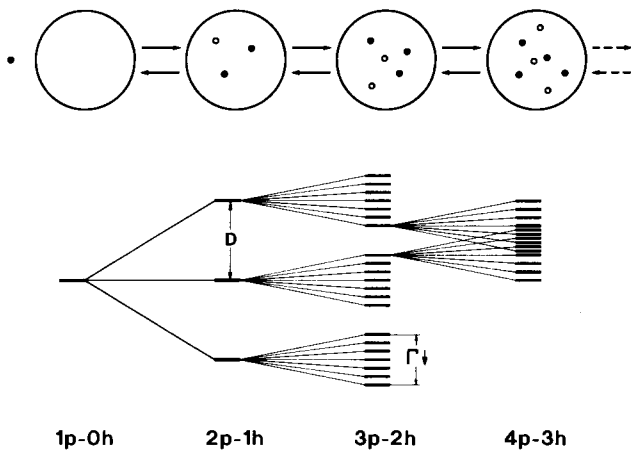


Fig. 4 Schematic representation of the increase in excited states against the number of collisions in the framework of the independent particle model.

With this assumption, after the second collision it may become possible to include also 3p-2h resonances to study the dynamics of nucleons in nuclei. We expect a group of 3p-2h resonances (almost equally spaced as the doorway resonances) which are interrupted by energy gaps depending on the difference in the level spacing and the spreading width. As shown in part 2 the spreading width for 4n target nuclei is small and a resonance spectrum discussed above has been observed in  $^{52}\text{Cr}+n$  data<sup>7</sup>. Their reduced width as a function of neutron energy is plotted in Fig.5. Two gaps are to be seen at 200 and 670 keV, where at least two resonances for each gap are missing.

A different presentation of these data is shown in Fig.6, where the relative level spacings  $D/\langle D \rangle$  are plotted against the order of occurrence ( $m$ ) of the resonance energies. The eye-guided lines for these values emphasize a periodical behaviour of the level spacings. The nearest spacing distribution for 18 values, excluding the gap values and those nearest to them obviously shifted by the repulsion effect, is shown in Fig.7. All spacings are limited to the interval  $0.5 \leq D/\langle D \rangle \leq 1.8$  for which the probability to be elements of a Wigner distribution is  $P_{18} = 0.12\%$ . The result favours a level distribution with a higher correlation between levels.

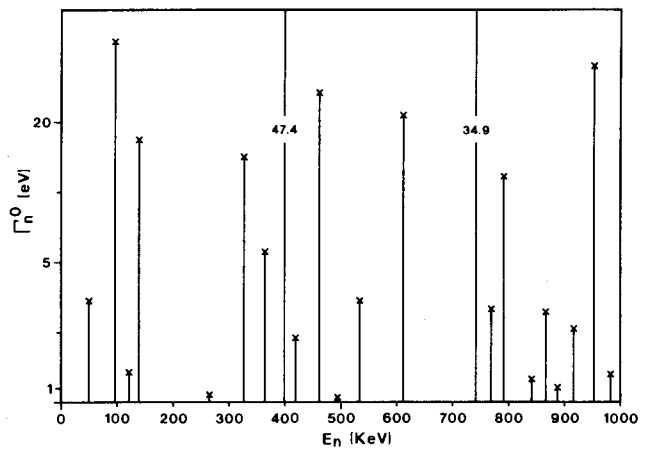


Fig. 5 s-wave resonance energies and reduced neutron widths for  $^{52}\text{Cr}+n$ .

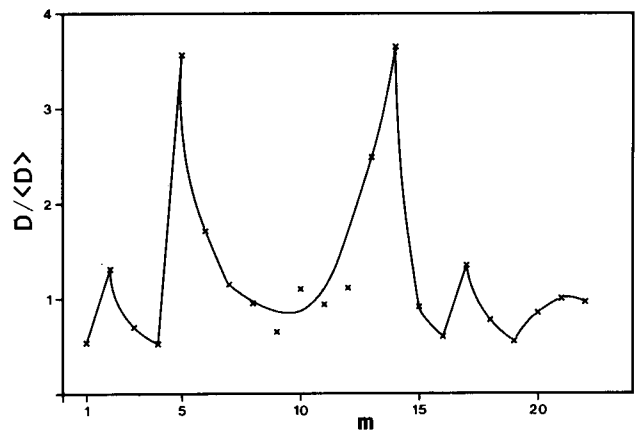


Fig. 6 The relative level spacings  $D/\langle D \rangle$  as a function of the order of occurrence of resonances.

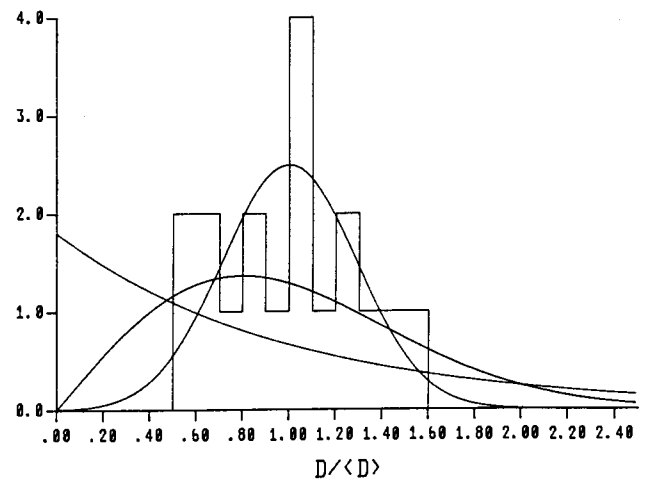


Fig. 7 Distribution of nearest level spacings for  $^{52}\text{Cr}+n$  resonances (18 spacings) compared with Poisson, Wigner and Gaussian functions.

### Resonance levels not specified

In this part resonance data are included which have similar properties described in the last two parts but they are taken from the literature (not measured at Geel) and are not assigned in respect of the number of collisions which would define the hierarchy of these states.

In Fig.8 the resonance parameters of  $^{40}\text{Ca}+n$  up to 750 keV are presented. The distribution of the 16 resonances obviously shows a strong repulsion effect. The level spacings are distributed in an interval  $0.56 \leq D/\langle D \rangle \leq 1.76$  with a probability of  $P_{15} = 0.41\%$  for obeying a Wigner distribution.  $^{40}\text{Ca}$  is the largest natural nucleus which has the same neutron and proton number.

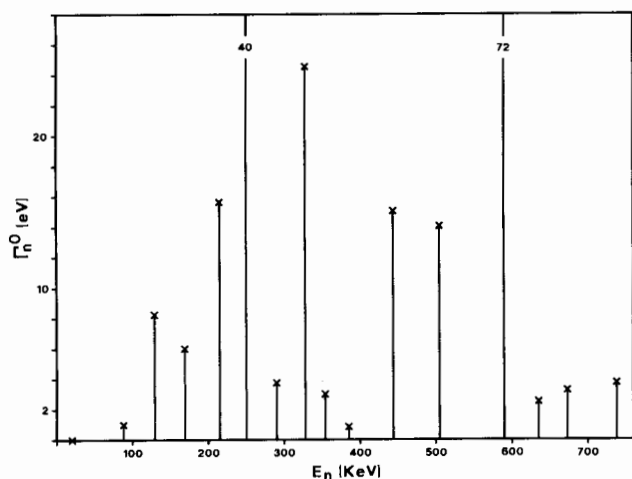


Fig. 8 s-wave resonance energies and reduced widths for  $^{40}\text{Ca}+n$ .

In Fig.9 the resonance parameters of  $^{96}\text{Zr}+n$  are plotted up to 100 keV. Seven spacings are distributed in the interval  $0.70 \leq D/\langle D \rangle \leq 1.66$ ; the probability that these values are distributed according to the Wigner distribution is  $P_7 = 1.86\%$ .  $^{96}\text{Zr}$  is the largest target nucleus with 4n nucleons where a strong energy correlation is observed. The probability for a Wigner distribution for both target nuclei  $^{40}\text{Ca}$  and  $^{96}\text{Zr}$  is  $P_{22} = 0.63\%$ .

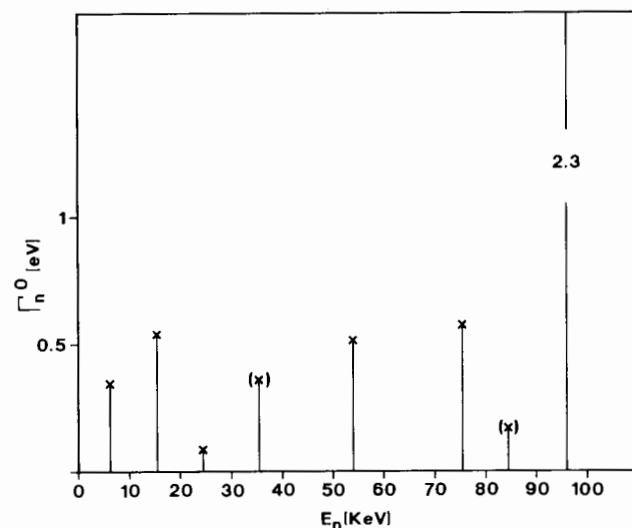


Fig. 9 s-wave resonance energies and reduced widths for  $^{96}\text{Zr}+n$ .

The selection of this type of resonance data from the literature certainly is not complete (see for instance Ref.8) and more resonances can be obtained by extending the neutron energy in transmission measurements, particularly for 4n target nuclei.

In summary a total of 50 spacings for five different isotopes have been studied where no small spacings are observed. A distribution of 42 spacings for the target nuclei  $^{40}\text{Ca}$ ,  $^{52}\text{Cr}$  and  $^{96}\text{Zr}$  are shown in Fig.10 which cover the interval  $0.55 \leq D/\langle D \rangle \leq 1.8$ . The probability that these spacings are elements of a Wigner distribution is  $P_{42} = 1.1$  ppm. The result supports a much stronger correlation of nuclear levels than described by the Wigner distribution. The doorway resonances and all 3p-2h resonances originating from the same doorway state are almost equally spaced.

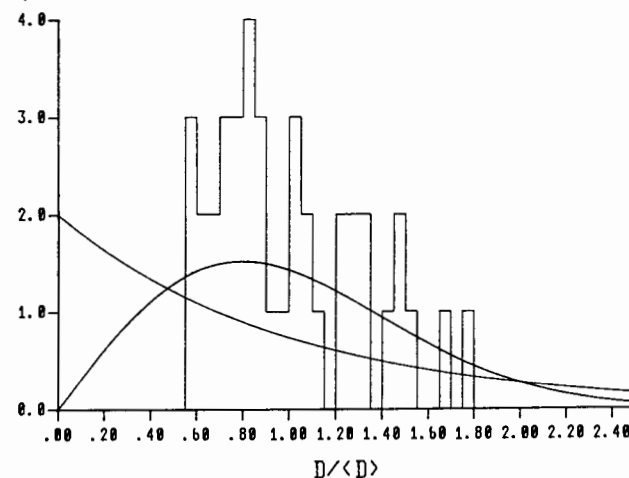


Fig. 10 Distribution of nearest level spacings for  $^{40}\text{Ca}+n$ ,  $^{52}\text{Cr}+n$  and  $^{96}\text{Zr}+n$  resonances (40 spacings) compared with Poisson and Wigner functions.

### Conclusions

The study of the simple excited resonances is fundamental for the study of nucleon-nucleon interaction in nuclei. In contrast to the classical doorway theory<sup>9</sup> which is derived independently from the type of interaction (or assumes electromagnetic-interaction), the experimental data for the nucleon-nucleon interaction give different results:

1. The position of doorway states in resonance spectra are indicated by discontinuities in the level spacing and not by the width of resonances.
2. The fragmentation and the spreading widths of doorway states at the threshold for a higher hierarchy of states are small and both increase monotonically with energy.
3.  $\Gamma^* \propto D^2$  indicates a repulsion effect and it is due to the nucleon-nucleon interaction.
4. Strong energy correlation between doorway resonances and resonances originating from the same doorway state results in a Gaussian-like nearest level spacing distribution.

The last two points indicate independently a mutual repulsion effect of resonances in contrast to the independent particle picture. A coupling between nucleons in a nucleus resulting in a strong correlation of the locations of the

nucleons may be responsible for the almost equal spacing. Since this coupling apparently does not change effectively the level spacing of states it could be treated as a residual interaction in the framework of the independent particle model. However a full understanding of this effect may be only expected from a sub-nucleonic particle interaction.

The importance of the almost equally spaced doorway states for the application is the fact that it fulfils in place of the single particle states the rather simple assumption of the Bethe level density formula. In addition, a smooth change in the level density with A is to be expected. Both aspects, and point 2, can explain the steps in the level density systematics. Also the level density parameter "a" can be described by a base line, which defines "a" for nuclei without shell effects and without residual interaction. Deviations from the base line are caused by including these effects and it becomes possible to determine the shell effect and, for instance, the pairing correlation for excited nuclei. Examples are given in Refs. 3, 8 and 10. The interpretation of all the deviations from the base line as a function of N and Z will make it possible to predict level spacings for nuclei which are not measurable.

## REFERENCES

1. H.A. Weidenmüller: Comments on Nucl. Part. Phys. 16, 199(1986) and references cited therein
2. V.F. Weißkopf: Phys. Today 14, 18(1961)
3. G. Rohr: Z. Phys. A318, 299(1984)
4. G. Rohr: Capture Gamma-Ray Spectroscopy 1987, Leuven, 643(1988), and to be published in Journ. of Nucl. Phys. and references cited therein
5. S.F. Mughabghab, M. Divadeenam and N.E. Holden: Neutron Cross Sections, Vol.1 Part A(1981) and Part B(1984)
6. H. Weigmann, P.W. Martin, R. Köhler, I. Van Parijs, F. Poortmans and J.H. Wartena: Phys. Rev. C 36, 585(1987)
7. G. Rohr, R. Shelley, A. Brusegan, F. Poortmans and L. Mewissen: to be published
8. G. Rohr, R. Shelley and G. Vanpraet: contribution to this Conference.
9. A. Mekjian: Advances in Nucl. Phys. 7, 1(1973)
10. G. Rohr: Capture Gamma-Ray Spectroscopy 1987, Leuven, 461(1988), and to be published in Journ. of Nucl. Phys.